Engineering Notes

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Performance Optimization of an Air-to-Air Missile Design

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I. Introduction

AN important part of the design process is the construction of a meaningful merit function to evaluate and compare different design alternatives. A useful merit function should include in it the salient design of an air-to-air missile, which is quite complex by nature. The merit function that is used is sometimes selected because it is easy to calculate, whereas a more appropriate one that better represents the design goal is rejected because of the difficulty and cost of calculating it. The natural design goal of a missile is performance, i.e., the capability of hitting a target from a large number of launch conditions. This Note presents the construction of a merit function for an air-to-air missile that is based on performance. Its application in the optimization of the missile design is shown. The specific application of the merit function is, together with a Davidon variable metric conjugate gradient optimization technique, to find an optimum set of design variables of a fictitious missile design. The Note discusses the merit function and the characteristics of the optimization.

II. Merit Function

A merit function that includes the salient design features plays a vital role in the comparison and evaluation of design alternatives. On the one hand, it should express completely the design goal, and, on the other hand, it should be simple and easy to evaluate, as a great many evaluations may be required in judging alternatives. The tradeoff between these diametrically opposite requirements (of ease of calculation and of completeness) is conducted based on time, money, and the need for a high-quality answer. On the air-to-air missile design problem, merit functions have been used that were based upon maneuverability considerations such as minimum radius turn, minimum time of flight, or worst-case conditions such as a particular difficult launch condition against a particular target maneuver. Such merit functions result in designs that are highly biased by the merit function and may not achieve the real design goal. For air-to-air missiles a primary design goal is performance, as evidenced by a capability to achieve a hit against a maneuvering target from a large number of different launch conditions. The problem may be quantified by selecting a finite set of launch conditions, a set of a few types of maneuvering targets, and searching over a restricted launch envelope, an example of which is shown in Fig. 1. In the illustration, the launch field was taken as a plane at a given altitude. The planar problem was expressed by selecting 5 angles of line of sight (LOS) radii, σ. For each radius, three directions of launch were considered, along the LOS, and $\pm 30°$ from it. Thus, three launch angles for five LOS angles gave a total of 15 launch conditions. Two types of maneuvering targets were considered, thus giving a 30-point quantification for the measure of missile performance. The merit function was calculated by finding (for each of these 30 cases) the minimum radius R of launch that results in a hit. This approach to quantifying the launch envelope is representative of many possibilities of measuring the overall performance of the missile by quantifying. The merit-function approach also can include the worst-case approach within it. Extension of this approach to a three-dimensional space is obvious. The particular choice of LOS angles and launch angles should, of course, be tailored to the exact design requirements of the specific missile being studied. The choice made in the foregoing applies to a fictitious missile.

As a further limitation of the merit function, the search for minimum radius successful launch was conducted only between 2900 m and 500 m. The merit function YP was

$$YP = \sum_{i=1}^{n} W_i R_{\min_i}$$

where $R_{\min_i}$ is the minimum successful (hit) launch radius for initial condition i, W_i is a weighting factor utilized to emphasize certain more critical launch conditions over others that are less critical, and n is the number of points (30 in the illustration). Furthermore, the resolution of the search was 100 m. The maneuvers of the target in the example were either an 8-g pull toward the missile, or a continued flight in a straight line at constant speed. The selection of a set of maneuvers in an actual design should, of course, be a function of the specific requirements. The merit function was programmed on a CDC Cyber 73 computer (using the missile simulation as a subroutine). Operation for one target maneuver, that is, $n = 15$, took 230 sec of computer time.

III. Optimization

Within the package of analysis tools, two programs utilized the merit function as a subroutine. One was an optimization program, the other a sensitivity program.

The optimization program searched for an optimum set of design parameters. In the illustration, the parameters of the guidance laws were selected for optimization; however, the program may be used for optimizing any set of parameters that are modelled in the missile system simulation, or any

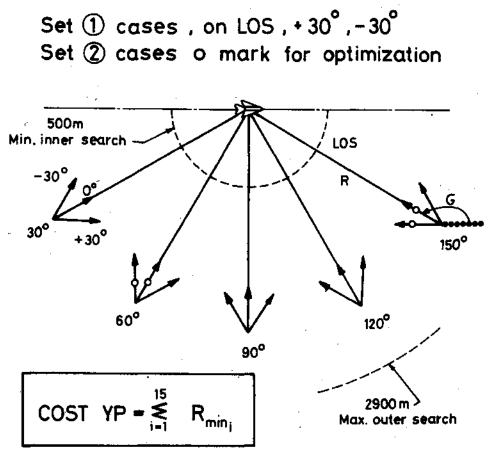

Fig. 1 Sample launch conditions.

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function of these parameters. Candidates might include weight, thrust, control limits, sensor accuracy, or even cost to build. If one considers a hyperdimensional space S whose coordinates are the set of the parameters of the system to be optimized, then there is a merit value associated with each point in the space S. There are also regions in this space where the merit is a maximum (in our illustration 15 initial conditions (IC's) $\times$ 2900 m maximum search limit = 43,500) and a number of points or regions where the merit is a minimum. The optimization problem is to search this parameter space to find the minima. Because of the nature of the design of an air-to-air missile that includes control limits, sensor limits, thrust limits, burn time limits, etc., there will be a large number of factors that interact to contribute to a failure or success. This may cause a number of local minima to occur, as will be seen in the illustration. In the example, a search for the global minimum was not conducted because the design goals do not require knowledge of the global minimum. An engineer confronted with several near-optimal designs can select an appropriate one based on sensitivity results, ease of construction, or other factors not included directly in the merit function.

The fact that local minima were suspected to exist led to the development of a two-step search procedure: 1) random research, and 2) Davidon conjugate gradient search. The random search was conducted, holding those parameters fixed for which it is believed a good starting estimate exists and varying the other parameters randomly. Running the random search program on the CDC Cyber 73 took 1500 sec for 3 parameters, 20 random searches. Once several low-cost candidate sets of parameters were found, the second optimization tool, the Davidon search, was performed. Estimation of derivatives for the Davidon algorithm was done by forward differences. The computer time required was 1800 to 3600 sec.

In operating the optimization program in the illustration, a subset of the merit function set of IC's was used. Four points instead of 15 were used to represent the launch envelope IC's for each maneuvering target. Using the entire set of 15 points would have involved prohibitive computer costs and would have been wasteful, as it was determined that a subset of four adequately represented the entire envelope. This is described in the following application. Once an optimum or near optimum design was found, sensitivity of the design was calculated, using the merit function as a subroutine. The sensititivity calculation took 460 sec of computer time for a double variation (e.g., $\pm 10\%$) of a single parameter.

IV. Application

The preceding tools were applied to a fictitious missile design. The air-to-air missile was a thrust vector guidance missile with fixed thrust and time of burn. The system included an onboard computer and a tracking sensor (that locks on to the target before launch). No sensor error models were included in the simulation. A fundamental problem in air-to-air missile guidance design is the tradeoff between making a tight turn, thereby reducing the velocity very much, or making a less tight turn and maintaining a fairly high velocity. In the first instance, the velocity may fall so low that the missile cannot catch up with the target after the turn is accomplished. In the second instance, the turn may be so slow that the missile sensor will lose track of the target or make such a long turn that it cannot catch up to the target before all the fuel is consumed. In the design studies, three guidance laws were assumed to exist: Launch Attitude Hold (LAH), Proportional Navigation (PN), and something akin to Pure Pursuit (PP). Switching occurs from the PN law to the PP law when the velocity of the missile falls below the value of the design parameter VSW (switch velocity of guidance low logic).

The selection of an appropriate subset from the 30 possible IC's was as follows. Chosing one type of target maneuver left a choice of 15 IC's shown in Fig. 1. A test set of four was selected, and optimization was done. The resulting optimum design parameters then were run in the complete merit function. If the overall change (improvement) paralleled the change achieved by the subset, then this subset was deemed suitable. If the change did not parallel the improvement achieved by the subset, a new subset was selected and the process repeated. In all, three sets were studied. Two sets are indicated in Table 1. A study using various weighting factors W_i in the merit-function optimization subset showed no improvement in overall design and a unit weighting was used throughout.

Table 1 Cost function initial conditions

Line of sight angle	Attitude angle	Comments[a]
150	150	subset A, B
150	120	
150	180	subset A, B, C
120	120	subset C
120	90	
120	150	subset C
90	90	subset B,C
90	60	
90	120	subset B
60	60	subset A
60	30	
60	90	subset A, C
30	30	
30	0	
30	60	

[a] Subset C was used for the random search.

Table 2 Changes in guidance law parameters

	PP1	PP2	PN1	PN2	VSW	Cost
Nominal	.20	.020	.000870	.0710	280	16,600
Set E	.23	.025	.000880	.0852	409	12,000
Set F	.30	.050	.000940	.0994	414	11,900
Set H	.30	.050	.002149	.2670	386	11,800

After operating the random search (using set C, Table 1), and conducting some linearized analysis at the best points, the Davidon technique was employed. The total region of successful launch conditions is marked by the inner contour of minimum launch radii, and an outer contour of maximum launch radii. At first it was found that this launch region envelope was continuous. However, as successive optimization was done, it was found that holes appeared in the launch region. There were isolated unconnected areas within the overall inner and outer radii marked region, where for a period of 200 to 500 m, successful launch could not be achieved even though at a closer or more distant radius success was achieved. This indicated the need to back off from the best optimum, if a continuous region was required (as is the case). In addition to this feature, many multiple local minima were suspected to exist; three were found and are shown in the results.

The results obtained using the merit cost program and optimization for a nonmaneuvering target show an improvement of 29% in cost. The change in guidance law parameters are shown in Table 2.

The sensitivity of the results was studied for all the guidance law parameters as well as thrust, nozzle time constant (TAU), nozzle angle maximum (δ_{max}), and look angle maximum (B_{max}). The results indicate a $\pm 10\%$ change in guidance law constants gave negligible change in merit. VSW showed a pronounced increase in merit in both directions at both minima. This fact indicated that it a sensitive parameter. It is of interest to note that the sensitivity at both minima F and H were very similar, although the minima are apart, in values of the guidance law parameters.